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Optimal Control of an EMU Using Dynamic Programming

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Abstract

A model is developed for minimizing the energy consumption of an electric multiple unit through optimized driving style, based on Hamilton-Jacobi-Bellman equation and Bellman's backward approach. Included are the speed limits, track profile (elevations), different driving modes and the train load. This paper includes aspects like the power loss in the auxiliary systems, time management, validation of the model regarding energy calculations and a study on discretization and the accuracy of the model. The model will be used as a base for a new driver advisory system.

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1. Introduction

To minimize the energy consumption of a train two main approaches are used: more energy efficient train units and energy efficient train operation i.e. improving the driving style of the train. This paper considers the second approach: the energy optimal control of train units, where the challenge is to plan the train movement during a trip with the aim to minimize the energy consumption. In the literature, several different algorithms have been used to solve the problem (see for example [1], [2] and [3]). The energy optimal driving style problem have also been studied in the automotive industry, mainly for applications to the heavy trucks (see for example [4] and [5]). In order to implement a solution for energy optimal train control on a real train it should ideally be implemented in the form of a system called driver advisory system (DAS), which gives suggestions to the driver on how to operate the train in an energy-efficient manner. Different driver advisory systems are available on the market. [6] and [7] offer a study on different DAS systems. Dynamic programming is among the widely used approaches to solve this kind of problems (see for example [1] and [8]) as it can deal better with constraints and functions such as local speed limits,

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running resistance and etc. [8]. A solution based on the Hamilton-Jacobi-Bellman equation and Bellman's backward approach is suggested in [9] to design a DAS system; this solution was further developed in [10]. This paper is a further development of [10], in which energy consumption of the auxiliary systems and the time management are added and where the model is validated and the accuracy of the model is estimated.

2. Train Model

The train is modeled as a single mass point. The tractive effort (F_t), the running resistance (F_{rr}) and the gradient force (F_g) are the forces affecting the train during the trip. The model is based on the following equation of motion:

$$m * a = F_t + F_{rr} + F_g, \quad (1)$$

where m is the mass of the train and a is the average acceleration. The energy consumed is equal to $F * x$ where x is the distance traveled and F is the force affecting the train (i.e. F_t when accelerating and $F_t + F_g$ when using the regenerative brakes). The train is equipped with regenerative brakes which regenerates 80 percent of the energy of the tractive effort and the gradient force back to the line.

3. Approach

The model is based on the Hamilton-Jacobi-Bellman equation and Bellman's backward approach, and was previously introduced in [9]. The different states of the train are represented using three discretized variables of time, distance and velocity. Time is the independent variable and the velocity and the distance are the state variables. The main idea is based on the Principle of Optimality [11] which can be stated using the following equation.

$$E(t_s, x_s, v_s) = \min_{v_f} \left(de(s, f) + E(t_s + dt, x_f, v_f) \right) ; \quad \forall v_f, \quad (2)$$

where dt is the length of one time step, $de(s, f)$ is the transition cost to reach from the state (t_s, x_s, v_s) (we call it the starting state) to the state $(t_s + dt, x_f, v_f)$ (we call it the final state) and $E(t_s + dt, x_f, v_f)$ is the minimum amount of energy needed to reach the destination from the state $(t_s + dt, x_f, v_f)$ (cost-to-go). The final state is chosen according to all the applicable velocities in the next time step (see [10]). x_f is calculated using x_s , dt , v_s and v_f , which lead to the following transition cost:

$$v_{avg} = (v_s + v_f)/2 \quad (3)$$

$$x_f = x_s + v_{avg} * dt \quad (4)$$

$$F_a = m * (v_f - v_s)/dt \quad (5)$$

$$F_t = F_a + F_{rr} + F_g \quad (6)$$

$$de(s, f) = F_t * (v_{avg} * dt) , \quad (7)$$

where the running resistance (F_{rr}) is calculated according to the equation presented in [9] and the gradient force (F_g) is the average of the gradient forces in all the distance steps in one transition. By backward iteration in time where the optimum cost at the final state is known, the optimum cost at each state can be calculated. Note that since $E(t_s + dt, x_f, v_f)$ in equation (2) is only available for a finite number of states based on the distance and the velocity steps, after the calculation of x_f , it should be rounded to the closest distance step. This will cause an error in the model which is discussed in section 6. It is also to be mentioned

that v_{avg} is a linear approximation which will affect the accuracy of the model.

4. Auxiliary system

It is assumed that a certain power is consumed for the auxiliary systems during the trip. The energy consumed for the auxiliary system (E_a) is equal to $P_a * t$, where P_a is the power and t represents the time passed. Therefore the transition cost will be $F_t * (x_f - x_s) + P_a * dt$. Since $d_e(s, f)$ is calculated during one time step and also because of the fact that P_a is constant during the whole trip, $F_t * (x_f - x_s)$ will be the determining factor in the transition cost. In other words adding the energy consumption of the auxiliary systems won't change the final solution. However since the model developed in this paper will also be used as a tool to check the energy performance of the trains, the energy consumption of the auxiliary systems is also added as an input in the model.

5. Time management

Time was fixed in the previous model introduced in [10], meaning that the train had to reach the destination at a certain time. In the model introduced in this paper though, it is possible to have the earliest and the latest arrival time for each trip. The arrival time constraints are enforced through assigning penalties to certain states. In the model introduced in [10] the destination state was $(T, X, 0)$ where T is the final time step and X is the final distance step. Any other state with X as their second parameter, had a high penalty for the model to avoid choosing them. In the model introduced here, the earliest arrival time ($T-EA$) and the latest arrival time ($T+LA$) are added. Penalties are assigned in ascending order to the states $(T_l, X, 0)$ where T_l is between T and $T + LA$. There are also penalties in descending order for the states $(T_e, X, 0)$ where T_e is between $T - EA$ and T . Assigning penalties in this order ensures that the priority is to have the trip time as close as possible to the specific planned arrival time. The penalties should be chosen carefully. With higher trip time the energy consumption will decrease, therefore if the penalties are not set properly, the model might always choose the latest arrival time. These penalties are smaller than the other penalties (e.g. non-zero velocity at the destination [10]). At the same time they are relatively big compare to the energy consumption of the train throughout the trip. In the current model, the common energy consumption is at most of the order of 10 power 6 or 7, whereas the penalties for the earliest arrival times are of the order of 10 power 10, the penalties for latest arrival times are of the order of 10 power 12 and the rest of the penalties are of the order of 10 power 19.

6. Results and Discussion

Figure 1 shows the optimal speed profile and the total energy consumption during the trip, for the state of $(0, 0, 0)$ in a trip of 1.5 km and 240 seconds. The dashed line represents the elevations (%) and the dotted line represents the speed limits. The optimal tractive effort needed at the moment is equal to 87.47[kN].

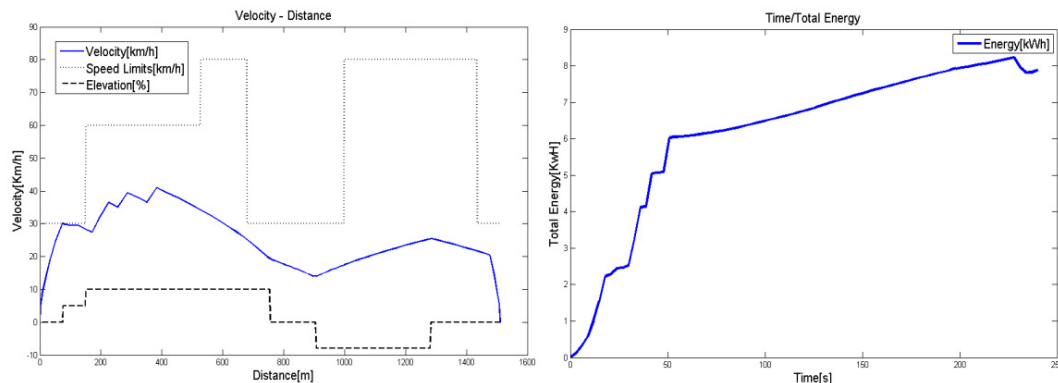


Figure 1-optimal speed profile and total energy consumption for the state of $(0,0,0)$

6.1 Validation

To evaluate the performance of the train, Bombardier Transportation is using a tool called TEP (Train Energy Performance). TEP has many features among which is the feature to simulate a trip of a train on a certain track and calculate the energy consumption and the performance of the train during the trip. To evaluate the energy calculations of the model, a trip 3.02 km was run both on TEP and the model developed in MATLAB. The trip time is 183 sec in TEP and 188 sec in the model developed in MATLAB. Table 1 includes energy calculations from TEP and the model in MATLAB.

Table 1-Comparison between TEP and the model in MATLAB

	TEP	MATLAB
Total Driving Energy[kWh]	16.92	17.43
Total Regenerated Braking Energy[kWh]	-12.50	-12.02
Net Energy[kWh]	4.42	5.41
Running Resistance Energy[kWh]	4.40	4.26
Auxiliary System[kWh]	14.73	15.13

Comparison shows that the driving energy in the model in MATLAB is 0.5 kWh more than TEP and braking energy is 0.5 kWh less. This is likely a result of discretization, as TEP uses the interpolation to calculate the different values for the tractive effort whereas in our model in MATLAB, tractive effort is calculated based on the time, distance and the velocity, hence the discrete tractive effort, which leads to the difference in energy calculations. Note that TEP has many features including a driver advisory system which was not used in this study. The comparison is only done for the validation of the energy calculations in the model and not the validation of the optimization technique. In the example here the trip consists of three phases of full acceleration, constant speed and full deceleration.

To examine the reduction in energy consumption the model is run with higher possible trip times. The results are available in table 2.

Table 2-Energy saving for higher trip times compare to the trip time of 188 sec

Trip time[s]	Total energy consumption reduction[%]
190	5.13
195	12.91
200	19.33
210	27.41
220	32.68

As it can be seen in the above table, the energy consumption will be significantly reduced with a slight increase in the trip time.

6.2 Discretization

Discretization in the approach presented in this paper causes some problems. One of the problems is the lack of coasting (i.e. driving the train without any tractive effort) in the final solution. Considering the fact that F_{rr} is a function of v_{avg} and F_g is a function of x_s , v_{avg} and dt (see [10], [9]), the equations (3-7) can be rewritten:

$$\Delta v = v_f - v_s \quad (8)$$

$$\Delta x = x_f - x_s = v_{avg} * dt \quad (9)$$

$$F_t = m * (\Delta v / dt) + F_{rr}(v_{avg}) + F_g(v_{avg}, dt, x_s) \quad (10)$$

$$d_e(s, f) = m * (\Delta v / dt) * \Delta x + \Delta x * [F_{rr}(v_{avg}) + F_g(v_{avg}, dt, x_s)] \quad (11)$$

Having F_t equal to zero in equation (13) will lead to the following equation:

$$m * (\Delta v/dt) = F_{rr}(v_{avg}) + F_g(v_{avg}, dt, x_s) \quad (12)$$

In the calculations of energy, F_t is calculated based on F_a , F_{rr} and F_g . F_{rr} is a relatively small amount, therefore having F_t equal to zero means that either F_g should be large, m should be small, or dv/dt should be small, which means smaller velocity steps and bigger time steps. Simulation results show that on a level track (no gradient force) of 3.02 km the biggest F_{rr} for a 184 tons train, is equal to 5.9 kN in one time step. Having equation (12) in mind, it can be concluded that coasting will only occur if dv/dt is lower or equal to 0.03 which is a small amount for short trips. As the result the solutions of the current model do not include much coasting, unless on a track with high altitudes (high F_g).

6.3 Calculation time

The main challenge in the approach used in this paper was to have the amount of calculation on the train as minimum as possible. As discussed in [10], the main idea is to have the optimal solution for all the states of the train beforehand. All the cumbersome calculations will be done before the trip and the calculations on the train will be only limited to looking up the solution at each state for the optimum decision [10]. However the calculation time for the general solution is still a problem. This is mainly because of the curse of dimensionality [12] which is the general downside of the Bellman's approach. In order to have a robust model, the length of each time, distant and velocity step should be constant in different journeys which makes the number of intervals increase in longer journeys and as a result the calculation time will increase rapidly. A potential way to decrease the calculation time is to use a different tool for programming. Currently the model is developed in MATLAB and implementing the model in a low-level language, such as C, would likely improve the performance.

6.4 Error

As presented before, at each time step, x_f is calculated according to the equations of motion and then it is rounded to the closest distance step. e_i is defined as the rounding error in each time step which is equal to $x_{fr} - x_{fi}$, where x_{fr} is the calculated distance step and x_{fi} is the rounded distance step at time $t_s + dt$. Total error (e_{total}) is calculated as the summation of the rounding errors in all the time steps. In practice, the rounding error is often manageable. As an example, for a trip of 3 km and 188 sec, using velocity and time steps of 40 and 5000 distance steps, the rounding error is 1.55 meters. This shows that at the arrival time, the train will be 1.55 meters ahead of the destination which is negligible considering the trip distance and the length of the train. The same error is calculated for different trip times. In this sample set (table 3), the e_{total} varies between -1.66 to 2.91. The root mean square of the total errors for the different trip times is equal to 1.51 meters which is negligible considering the length of the train. Moreover, to check the accuracy throughout the trip, the Root-Mean-Square-Error (i.e. $\sqrt{(\sum e_i^2)/T}$) is calculated. The results are available in table 3. Note that the results presented in the table 3 are just samples and not the absolute results as they may vary with different conditions (e.g. different trip time, elevation profile, different train data, etc.).

Table 3 - table of errors for each trip time in a 3 km trip with no elevations

Trip Time [s]	e_{total} [m]	RMSE [m]
188	1.55	0.15
190	1.11	0.19
195	2.91	0.14
200	0.55	0.16
205	-0.90	0.17
210	-1.66	0.15
215	1.04	0.18
220	1.11	0.17

7. Conclusion and Future Works

A model was introduced previously as a base for a driver advisory system for electric trains in [9] and [10]. In the model presented in this paper the energy consumption of the auxiliary systems is added. The model is also validated regarding energy calculations using the Bombardier's own tool and the accuracy of the model is estimated. Moreover a discussion is presented on the discretization of the variables in the model. There are several causes of energy loss in the train. So far the driving energy and the energy consumption of the auxiliary systems are added. There are power losses in different parts of the motor and the propulsion system which need to be included. Although the model is validated, there's still a need to validate the model using the real data. The model should also be validated regarding the minimization of energy consumption. Furthermore, in order to have the model on a real train, the calculation time should be reduced. It is to be mentioned that this paper is a part of an ongoing project and there are still works to be done to have the driver advisory system ready.

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References

- [1] Franke, R., P. Terwiesch, and M. Meyer. An algorithm for the optimal control of the driving of trains. 2000. *Proceedings of the 39th IEEE Conference in Decision and Control*. 2000. pp. 2123-2128 vol.3
- [2] Miyatake M, Matsuda K. Energy Saving Speed and Charge/discharge Control of a Railway Vehicle with On-board Energy Storage by Means of an Optimization Model, *IEEJ Transactions on Electrical and Electronic Engineering* 2009 4:771 – 778
- [3] Chang C.S, Sim S.S, Optimizing Train Movement through Coast Control Using Genetic Algorithms. *IEE Proceedings – Electrical Power Applications* 1997, 144(1):65 – 73
- [4] Hellström E, Ivarsson M, Åslund J, Nielson L. Look-ahead control for heavy trucks to minimize trip time and fuel consumption. *Control Engineering Practice* 17 (2009) 245-254
- [5] Hellström E, Åslund J, Nielson L. Design of an efficient algorithm for fuel-control look-ahead control. *Control Engineering Practice* 18 (2010) 1318-1327
- [6] DeltaRail Group, Driver Advisory Information for Energy Management and Regulation. In: Stage1 Report, RSSSB Research Program 2009, T724
- [7] Panou K, Tzieropoulos P, Emery D. Railway Driver Advice Systems: Evaluation of Methods, Tools and Systems. *Journal of Rail Transport Planning & Management* 2014, <http://dx.doi.org/10.1016/j.jrtpm.2013.10.005>
- [8] Ko H, Koseki T, Miyatake M, Application of dynamic programming to optimization of running profile of a train, *WIT*

transaction on the built Environment, 74, pp. 103-112. 2004

[9] Gkortzas P. Study on Optimal Train Movement for Minimum Energy Consumption, Master Thesis Work, Mälardalen University 2013. p. 82

[10] Ghaviha N, Bohlin M, Wallin F, Dahlquist E, an Algorithm for the Optimal Control of an Electric Multiple Unit, *SIMS 2014 – 55th international Conference on Simulation and Modeling*

[11] Bertsekas D.P. *Dynamic Programming and Optimal Control, third edition*. Athena Scientific. 2005, ISBN 1-886529-26-4

[12] Zhang H-G, Zhang X, Luo Y-H, Yang J, An Overview of Research on Adaptive Dynamic Programming, *Acta Automatica Sinica*, vol. 39, pp. 303-311, 2013



Biography

Nima Ghaviha received the B.S. degree in industrial engineering from Sharif University of Technology, Iran and M.S. degree in product and process development from Mälardalen University, Sweden. He is currently working toward getting his PhD degree at Future Energy Center at Mälardalen University, developing his research work in energy optimization of train transportation.